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Title: Berezinskii-Kosterlitz-Thouless Transition in Quantum Critical Metals

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Intended for: talk in Munich and Leiden

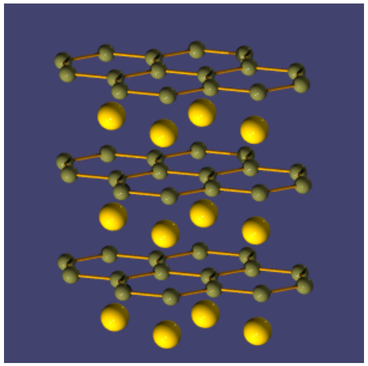
Berezinskii-Kosterlitz-Thouless Transition in Quantum Critical Metals

Jian-Huang She

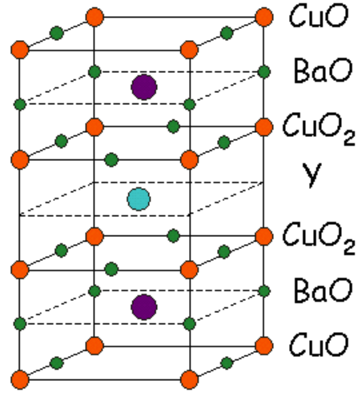
Los Alamos National Laboratory

JHS & A. V. Balatsky, Phys. Rev. Lett. 109,077002(2012)

Lower dimension, stronger correlation, higher Tc ?

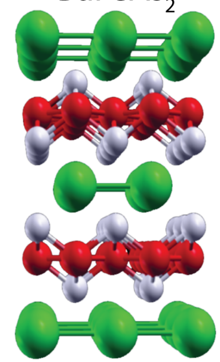


MgB_2: T_c=39K



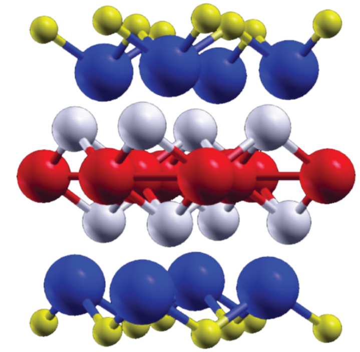
$T_c^{max} = 93K$

$BaFeAs_2$

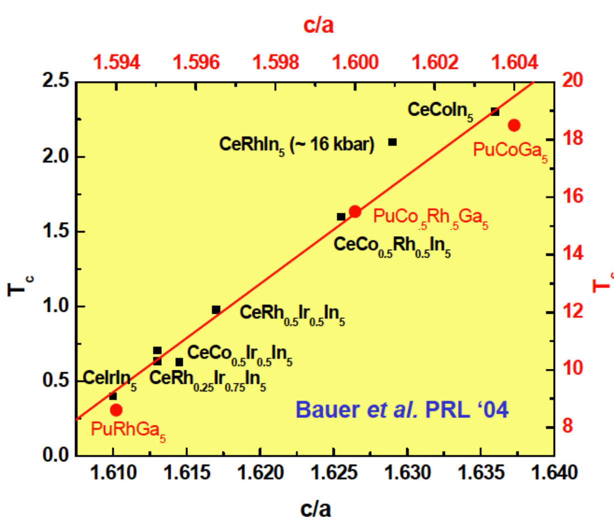


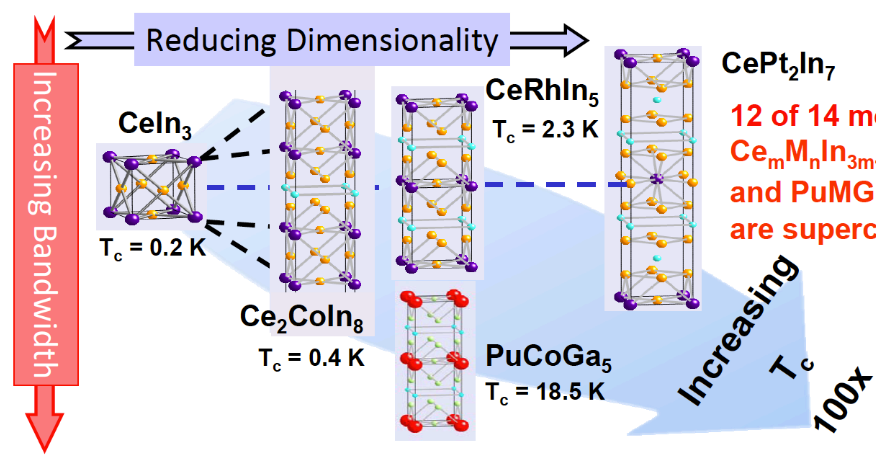
$Ba_{0.6}K_{0.4}Fe_2As_2$: 38K

SmFeAsO



$SmFeAsO_0.85$: 55K





12 of 14 members of $Ce_mM_nIn_{3m+2n}$ and $PuMGa_5$ family are superconductors

Eric Bauer

Lower dimension, more fluctuation, lower Tc ?

1, High dimension: mean field behavior (d > upper critical dimension)

2, Low dimension, more fluctuations (Mermin-Wagner theorem) :

continuous global symmetry can not be broken in 2d at finite T.

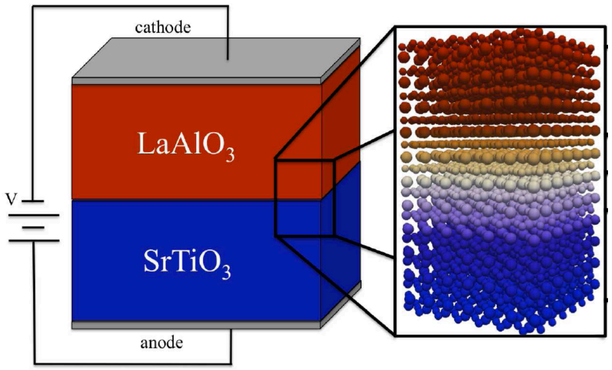
$$H \sim \int d^d x (\nabla \phi)^2 \qquad \langle \phi^2(0) \rangle \sim \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2}$$

d=1,2, the integral diverges at IR. (d=2: lower critical dimension)

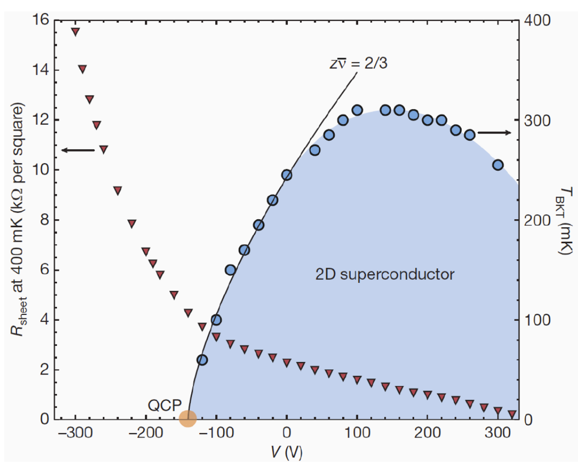
How to get high Tc (nature's solution): heterostructure

2 dimensional correlation, 3 dimensional phase transition

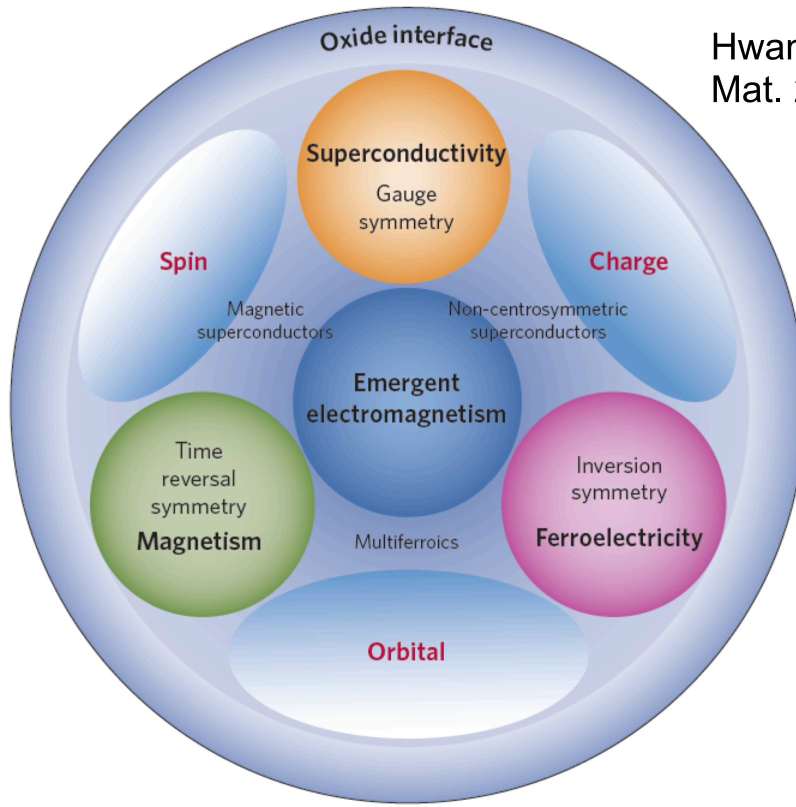
Strongly correlated heterostructures: oxide interface



Haraldsen, Wolfle, Balatsky, PRB 2012



Caviglia *et al.*, Nature 2008

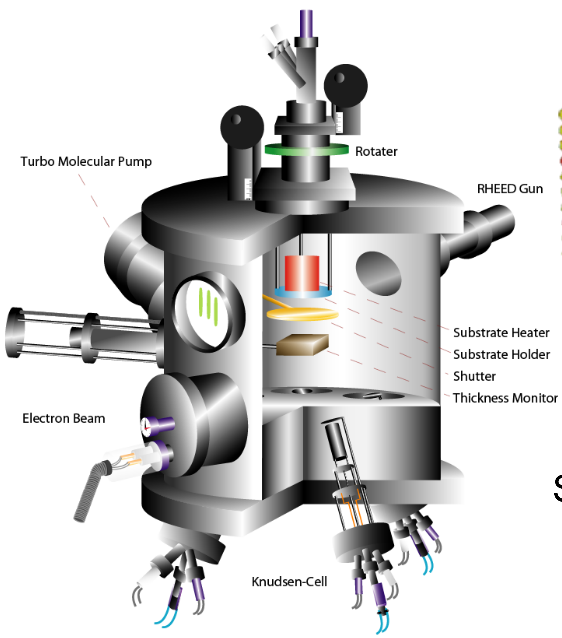


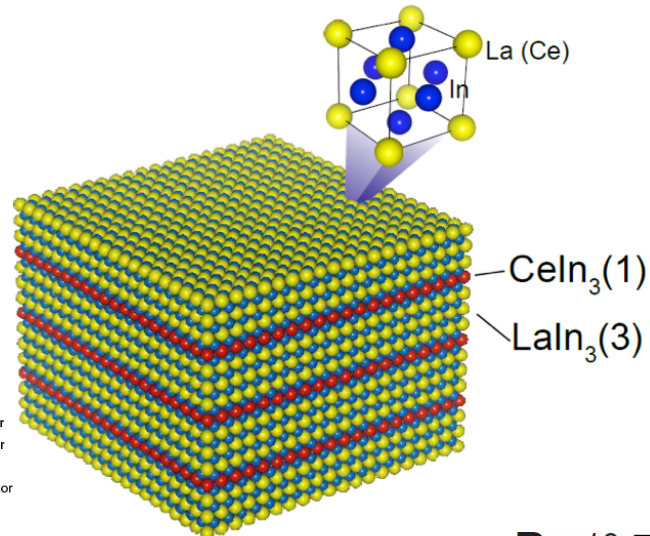
Hwang *et al*. Nat. Mat. 2012

How about f-electrons?

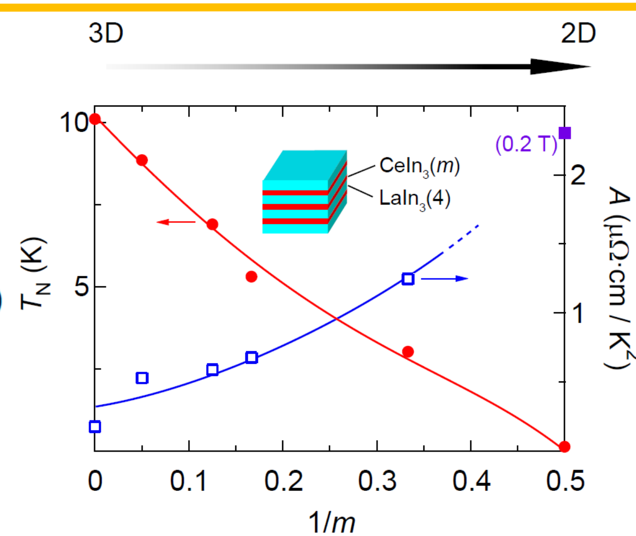
Heavy fermion goes two dimensional: CeIn3/LaIn3

Molecular Beam Epitaxy

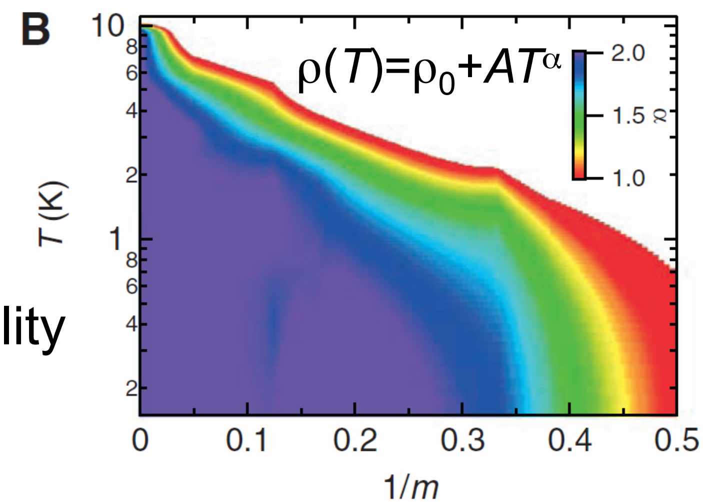




Shishido *et al*. Science 2010



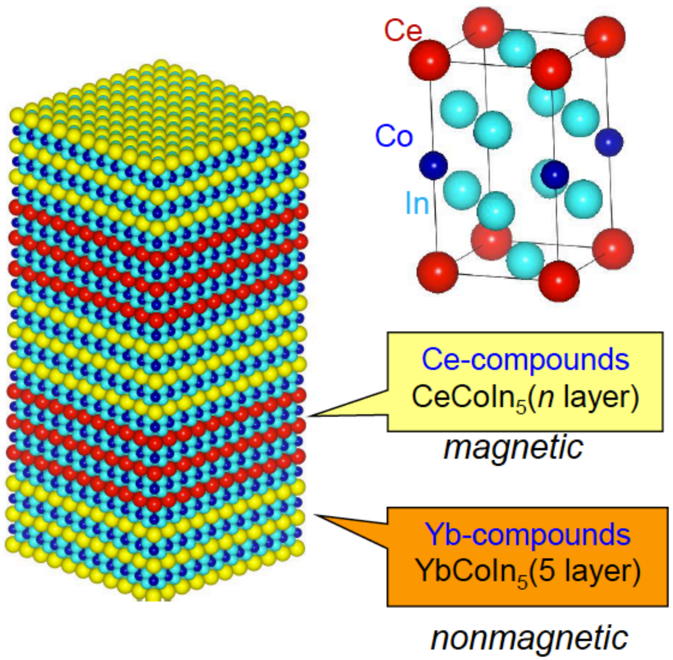
Dimensionally tuned quantum criticality



Heavy fermion goes two dimensional: CeCoIn5/YbCoIn5

Heavy-fermion superconductor

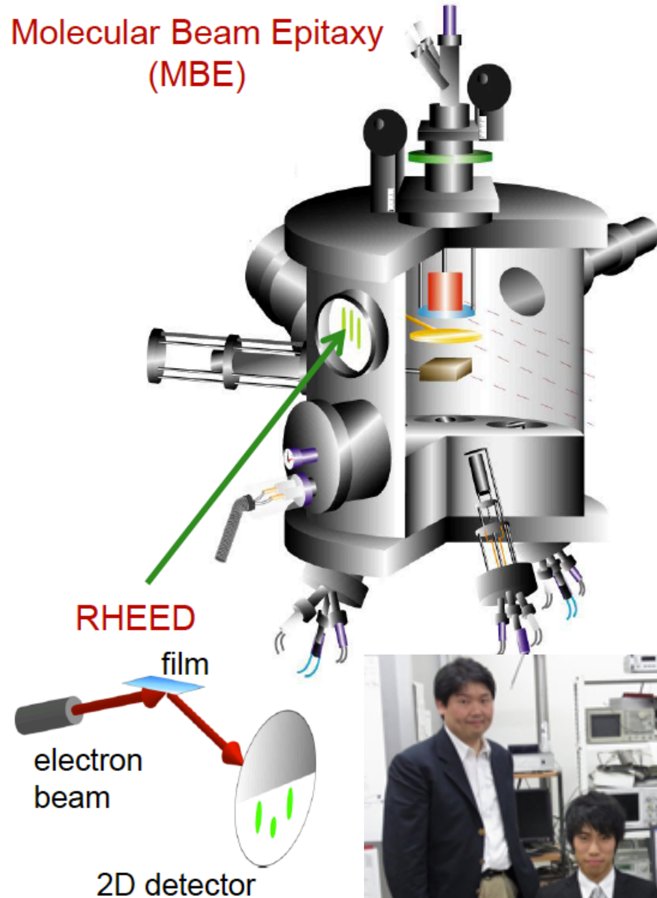
$CeCoIn_5$ (T_c=2.3K)

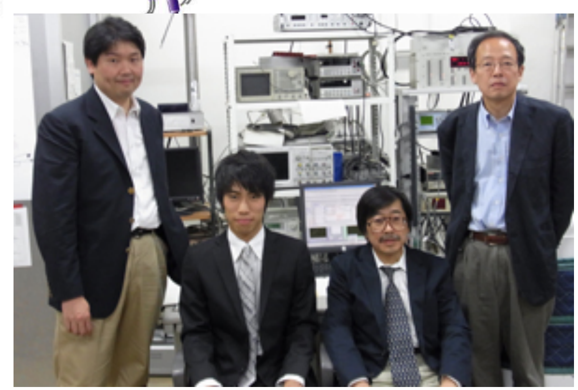


typically repeated 30~60 times
on the (001) surface of MgF_2

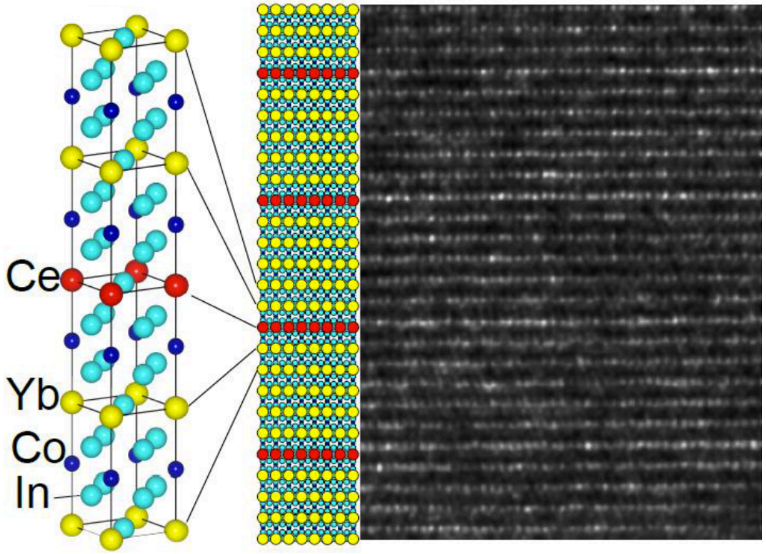
Mizukami *et al*. Nat. Phys. 2011

Molecular Beam Epitaxy
(MBE)

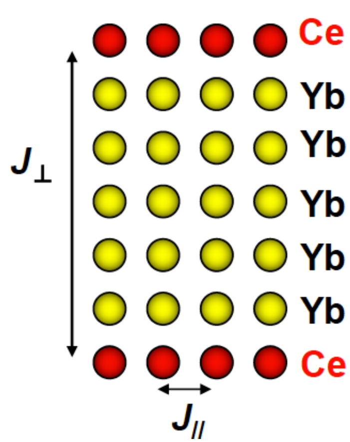




Two dimensional Kondo lattice

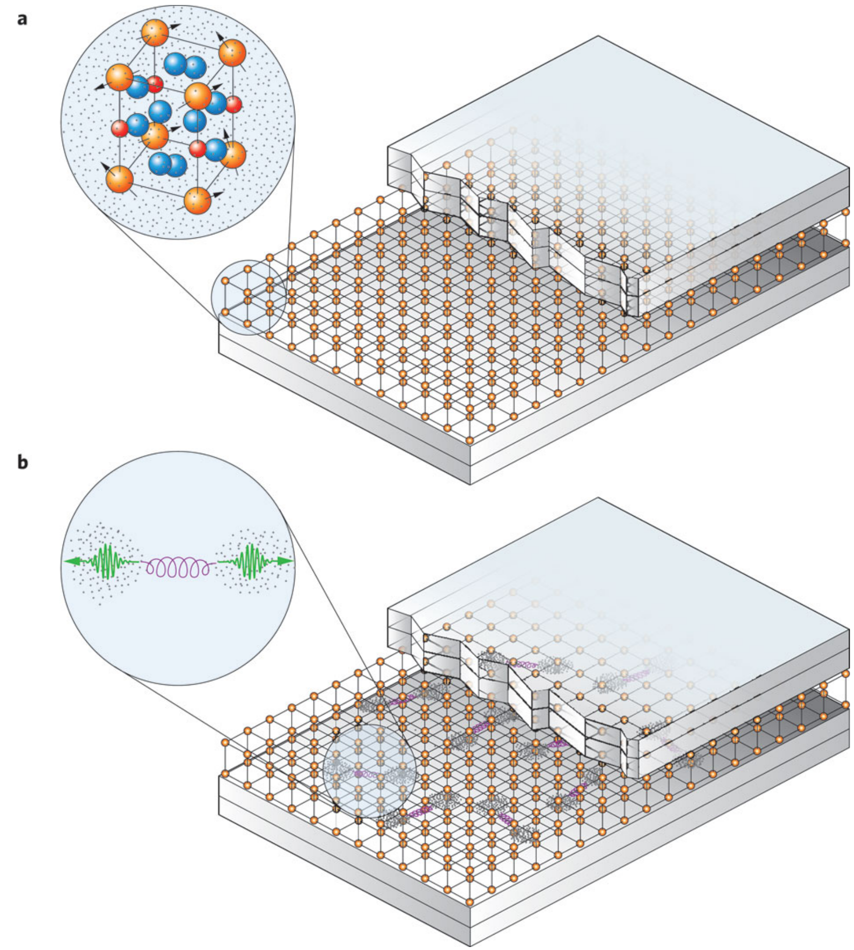


Mizukami *et al*. Nat. Phys. 2011



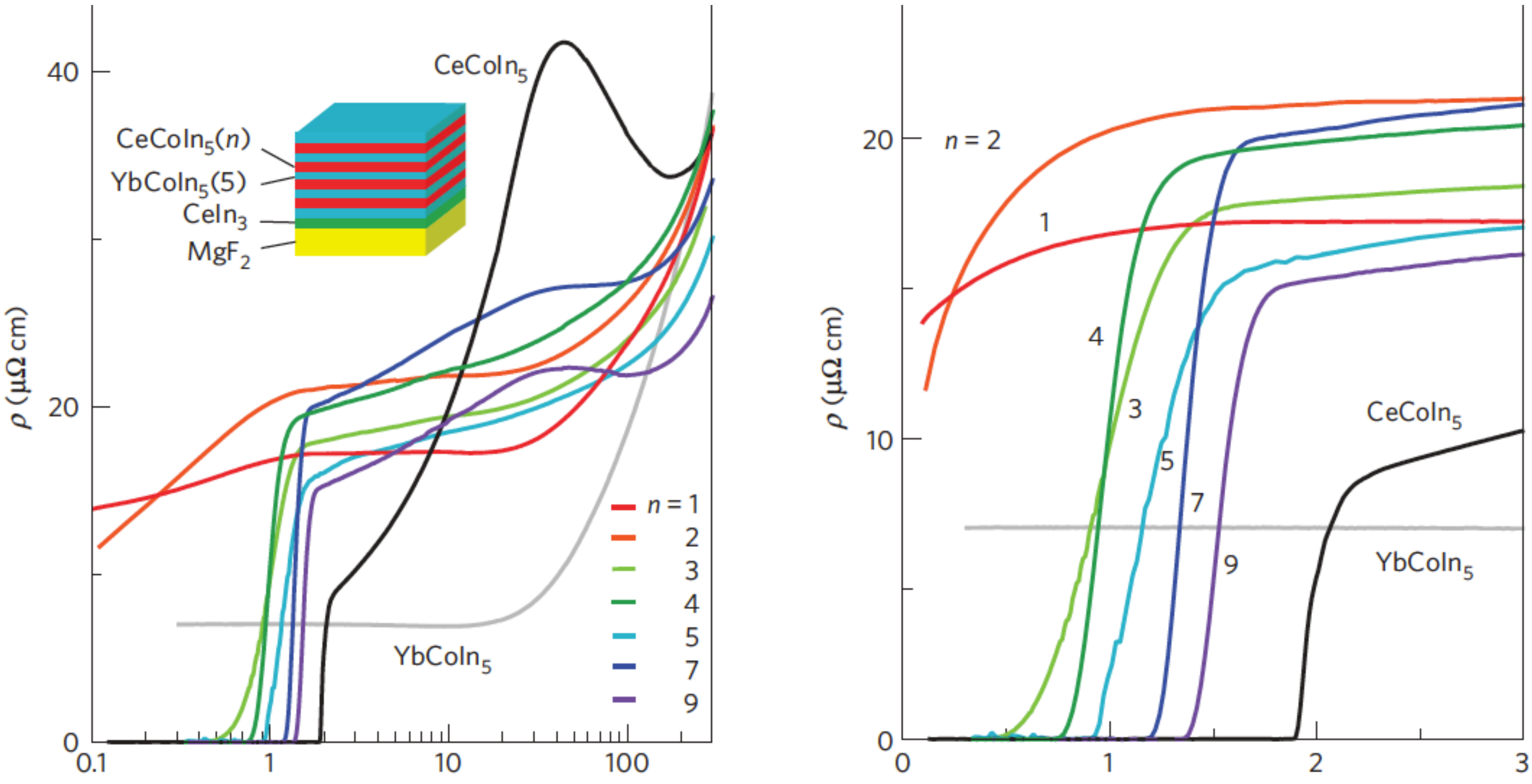
RKKY interaction $\sim 1/r^3$

$J_\perp / J_{//} \sim 0.1$ %

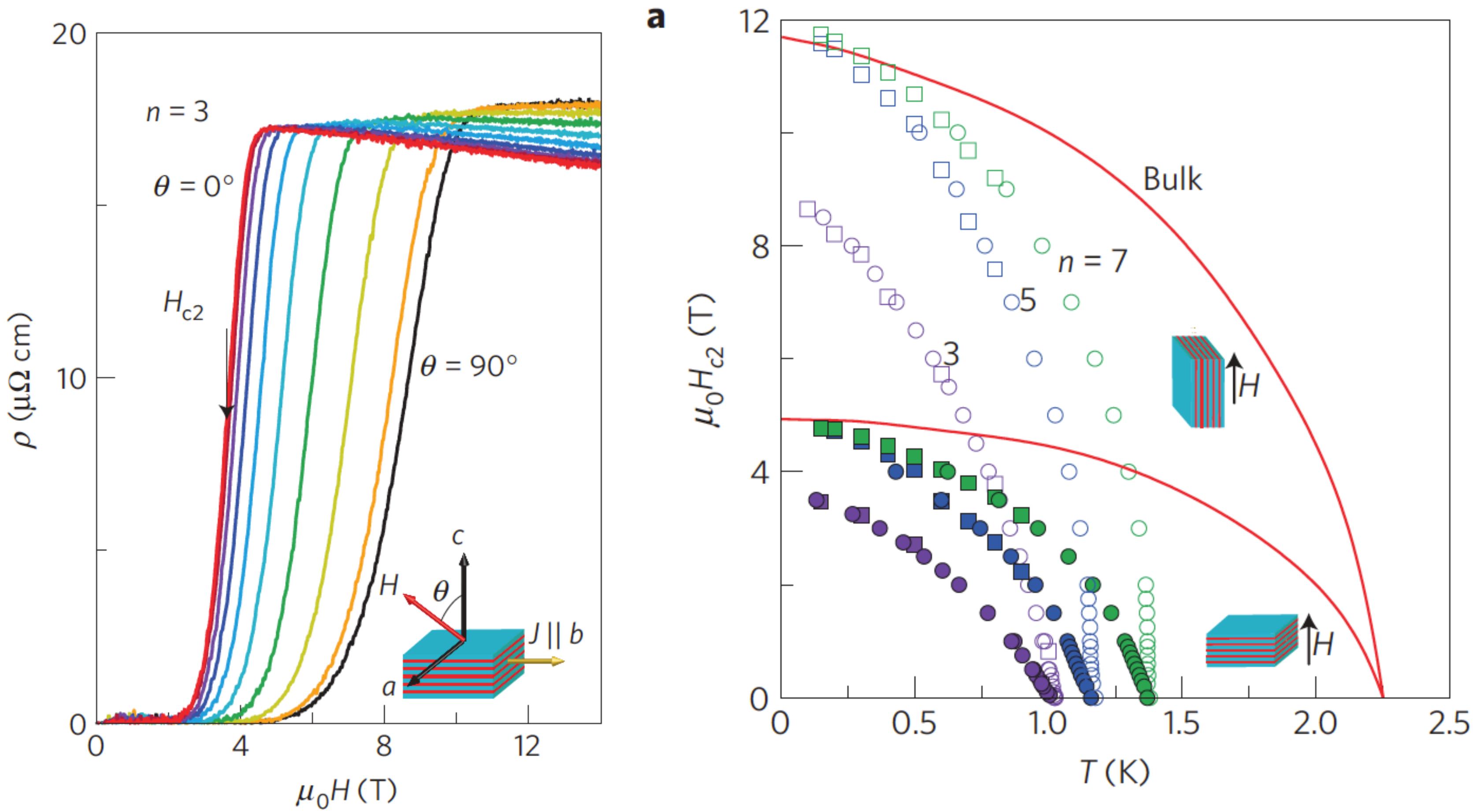


Thompson, Nat. Phys. 2011

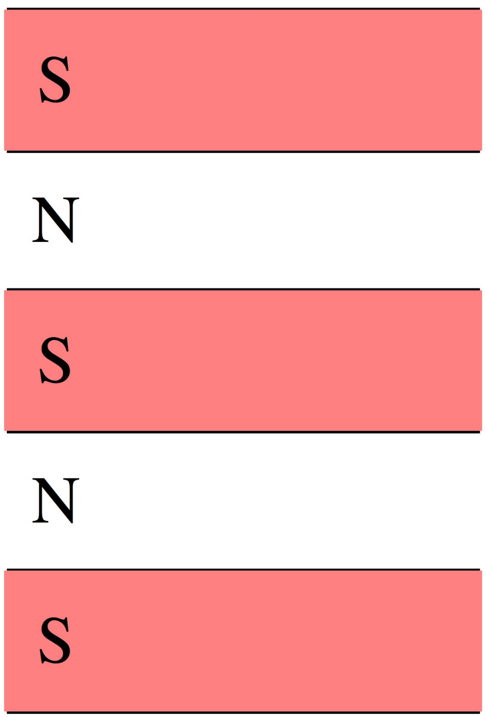
Superconductivity in HF superlattices

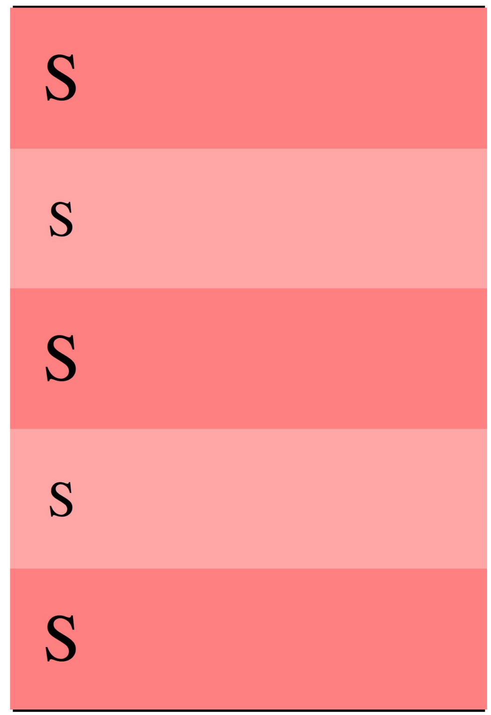


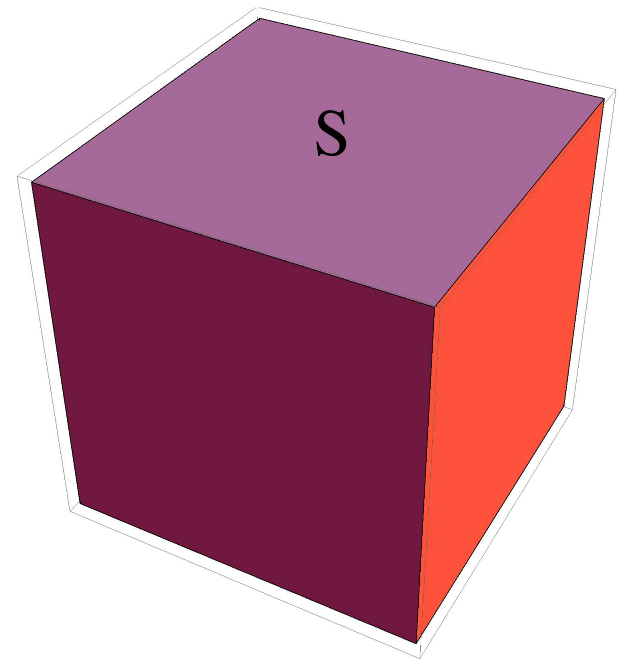
With magnetic field



2d or 3d: proximity effect







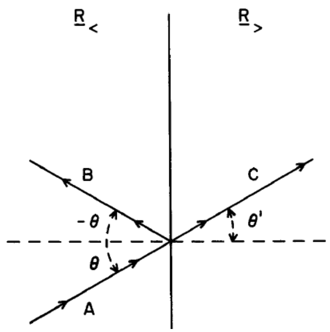
<u>Thickness of leakage region</u> much larger than <u>interlayer spacing</u>

$$l = \frac{\hbar v_N}{2\pi k_B T} \simeq 100\text{nm}$$

$$\text{d} \simeq 3.7\text{nm}$$

Suppressed proximity effect from mass mismatch

E. W. Fenton (1985)



Transmitted probability current

$$\lambda \approx \frac{4m_{light}}{m_{heavy}} \qquad m_h \gtrsim 10^2 m_\ell$$

Transmission: one percent !

Valls, Bryan, Zutic (2010)

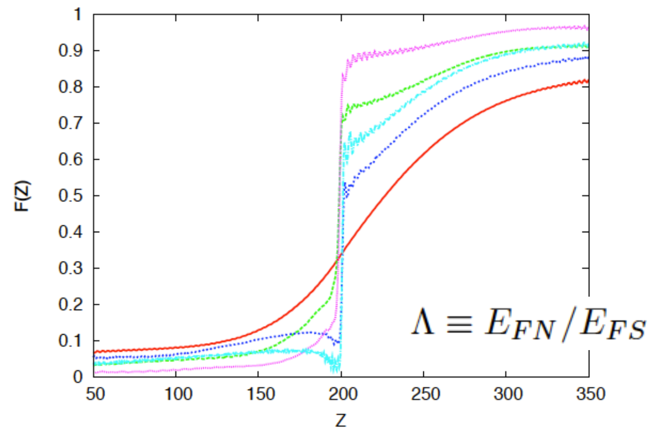


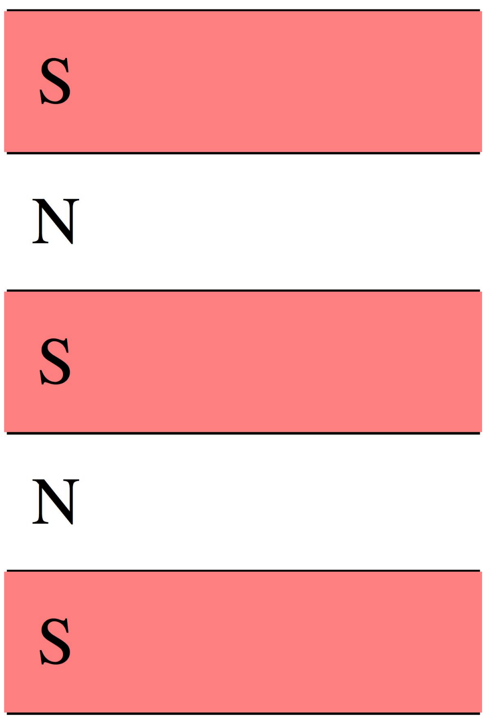
FIG. 3: (Color online) The spatial dependence of the pair amplitude $F(Z)$ for five different mismatch values Λ =1/4 (purple), 1/2 (green), 4 (cyan), 2 (blue), and 1 (red), from top to bottom on the right side. The results are given for $H_B = 0$, $g = -1/3$ at $T = 0.3T_c$.

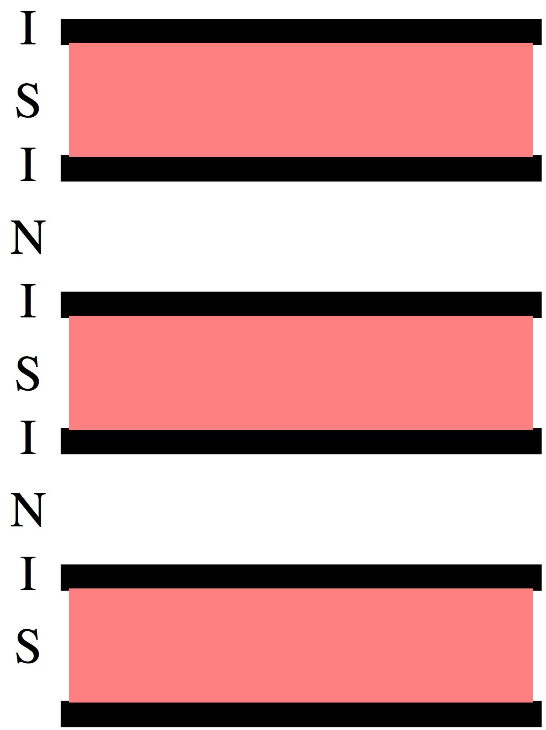
Effective barrier: $Z = (Z_0^2 + (1-r)^2/4r)^{1/2}$

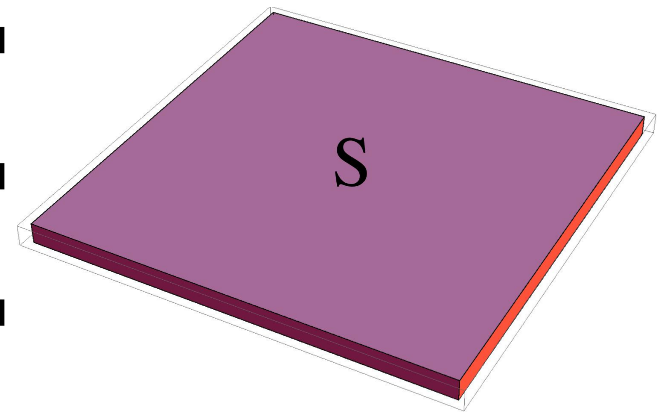
$$r = v_S/v_N$$

Blonder, Tinkham, Klapwijk (1982)

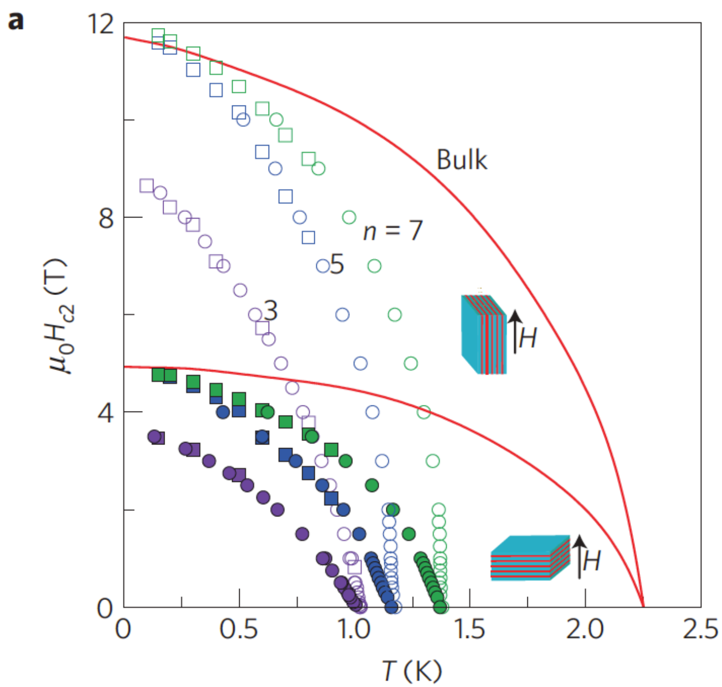
Two dimensional superconductor

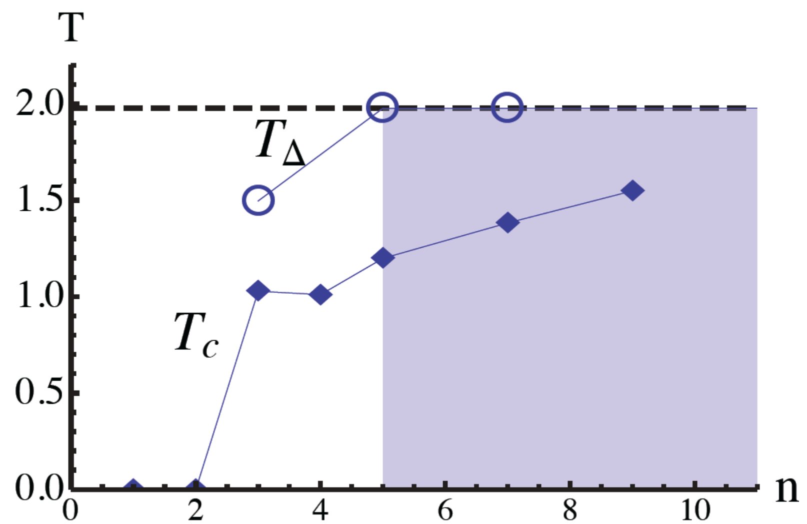






Gap and Tc





Pauli limited upper critical field

$$H_{c2}^{\text{Pauli}} = \sqrt{2}\Delta / g\mu_B$$

Phase fluctuations!

This is a Berezinskii-Kosterlitz-Thouless transition!

Consider a system with U(1) symmetry, e.g. XY-magnet, superfluid…

Landau: symmetry breaking, second-order phase transition

$$\text{As } |\boldsymbol{r}_1 - \boldsymbol{r}_2| \to \infty,\ \langle \Phi(\boldsymbol{r}_1)\Phi(\boldsymbol{r}_2)\rangle \sim \begin{cases} e^{-|\boldsymbol{r}_1-\boldsymbol{r}_2|/\xi}, & \text{for } T > T_c \\ \text{constant}, & \text{for } T < T_c \end{cases}$$

Mermin-Wagner: no broken continuous symmetry for d=2,1, due to strong fluctuations

BKT: quasi-long-range-order and true phase transition in 2d

$$\text{As } |\boldsymbol{r}_1 - \boldsymbol{r}_2| \to \infty,\ \langle \Phi(\boldsymbol{r}_1)\Phi(\boldsymbol{r}_2)\rangle \sim \left(\frac{|\boldsymbol{r}_1 - \boldsymbol{r}_2|}{r_0}\right)^{\eta},\ \text{for } T < T_c.$$

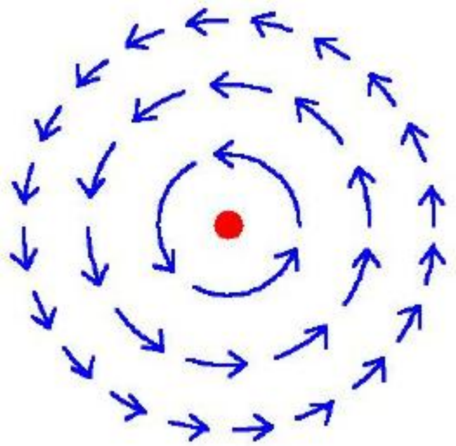
Vortex: topological defect in systems with U(1) symmetry

Superconducting order parameter: $\Psi(\mathbf{r},t) \equiv A(\mathbf{r},t)\exp i\phi(\mathbf{r},t)$

Decompose phase into smooth and singular parts:

$$\phi(\boldsymbol{r}) = \phi_{\rm sm}(\boldsymbol{r}) + \phi_{\rm sg}(\boldsymbol{r})$$

$$\oint d\boldsymbol{r}\cdot\nabla\phi_{\rm sm}(\boldsymbol{r}) = 0 \qquad \oint d\boldsymbol{r}\cdot\nabla\phi_{\rm sg}(\boldsymbol{r}) = 2\pi n$$



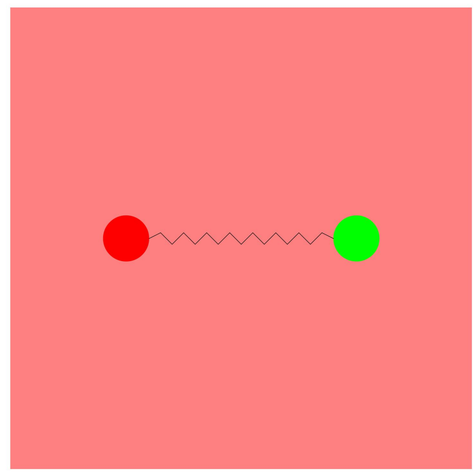
Vortex-vortex interaction

$$\frac{\mathcal{H}_v}{k_B T} = -\pi K \sum_{ij} n_i n_j \log \frac{|\mathbf{r}_i - \mathbf{r}_j|}{R_0} - \log y \sum_i n_i^2$$

Stiffness: $K = n_s \hbar^2 / 4 m k_B T$

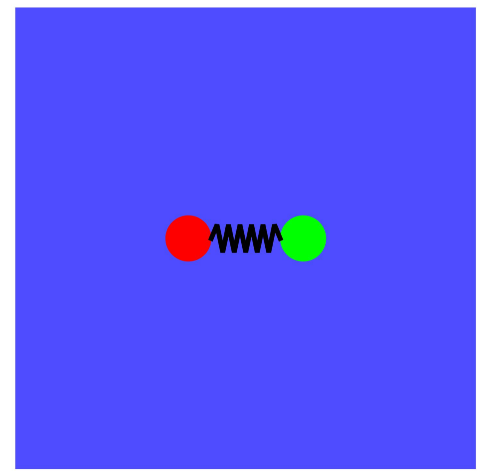
Fugacity: $y = e^{-E_c / k_B T}$

Defect mediated phase transition at finite temperature



High T

free vortex



Low T

bound pairs

Feature I: universal jump in superfluid density

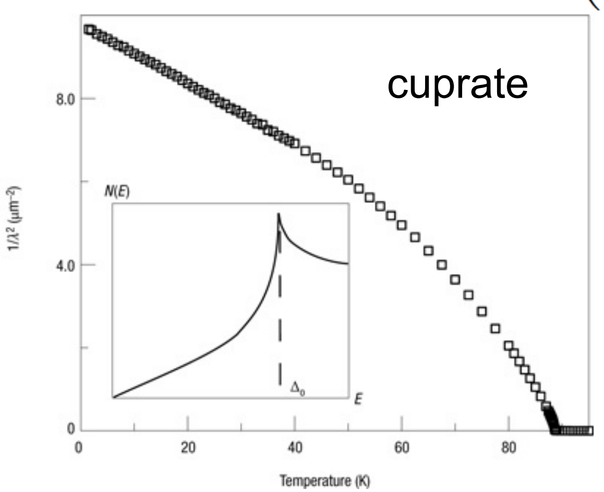
Kosterlitz's RG equation:

$$\begin{aligned}\frac{d}{dl}K^{-1}(l) &= 4\pi^3 y^2(l),\\ \frac{d}{dl}y(l) &= [2-\pi K(l)]y(l)\end{aligned}$$

Superfluid density has a jump at Tc:

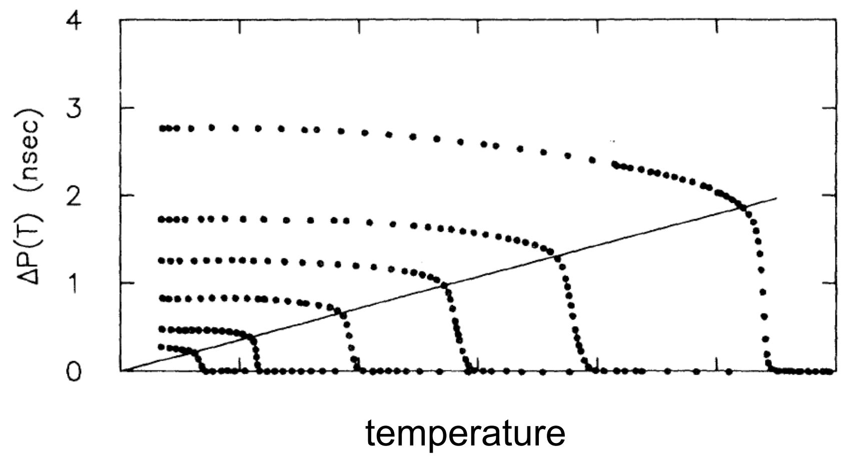
$$K \equiv \frac{\hbar^2 \rho_s(T)}{m^2 k_B T} = \begin{cases} 0, & \text{for } T = T_c^+ \\ 2/\pi, & \text{for } T = T_c^- \end{cases}$$

Mean field transition: $\rho_s(T) \sim 1 - \left(\frac{T}{T_c}\right)^\alpha$



Bonn, 2006

Superfluid helium film: $\Delta P \propto \rho_s$



McQueeney, Agnolet, Reppy (1984)

Feature II: thickness dependence of transition temperature

Universal relation: $$k_B T_{\mathrm{BKT}} = \frac{\pi \hbar^2 n_s^{2D}(T_{\mathrm{BKT}})}{8 m \epsilon_c}$$

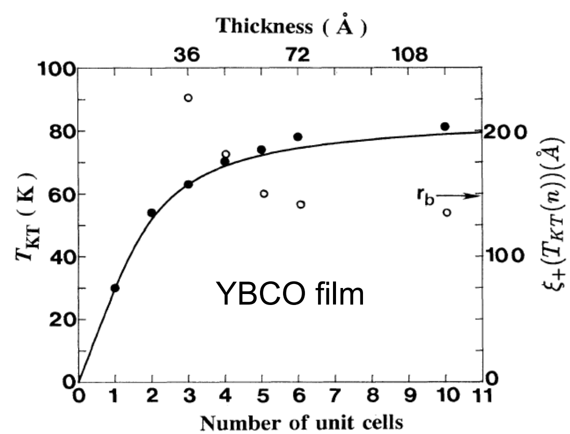
Carrier density: $$n_s^{2D} = n_s^{3D} d,$$

$$n_s^{3D}(T) = n_s^{3D}(0)\lambda_b^2(0)/\lambda_b^2(T)$$

Penetration depth: $$\lambda(T) = \frac{\lambda(0)}{\sqrt{1 - (T/T_{c0})^\alpha}}$$

Thickness dependence of BKT temperature:

$$\frac{T_{\mathrm{BKT}}}{1 - (T_{\mathrm{BKT}}/T_{c0})^\alpha} = \frac{\pi \hbar^2 n_s^{3D}(0)}{8 k_B m \epsilon_c} d$$



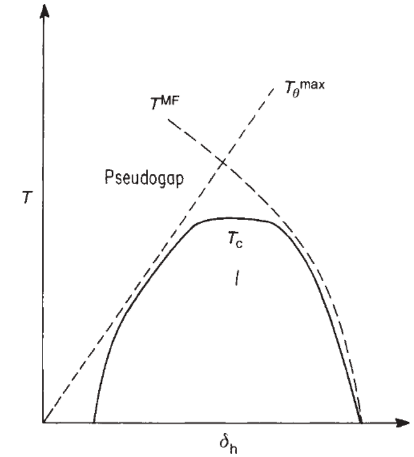
Matsuda *et al.*, 1993

Phase fluctuations in cuprates

Uemura relation (underdoped): $T_c = \mathrm{const} \times \rho_s(0)$

$$H = \tfrac{1}{2} m^* n_s(0) \int \mathrm{d}r \, v_s^2 \qquad v_s = \hbar \nabla \theta / 2m^*$$

$$T_\theta^{\max} = A V_0 \qquad V_0 = \frac{(\hbar c)^2 a}{16 \pi e^2 \lambda^2(0)}$$



Emery, Kivelson, 1995

Homes' law for cuprates (all doping): $\rho_s \propto \sigma_{dc} T_c$

Towards a holographic realization of Homes' law

Johanna Erdmenger, Patrick Kerner and Steffen Müller

Max-Planck-Institute for Physics, Werner Heisenberg Institut, 80805 Munich, Germany

Feature III: temperature dependence of resistivity

Above BKT temperature, there are unpaired vortices, which give rise to resistivity.

Kosterlitz's RG equation:

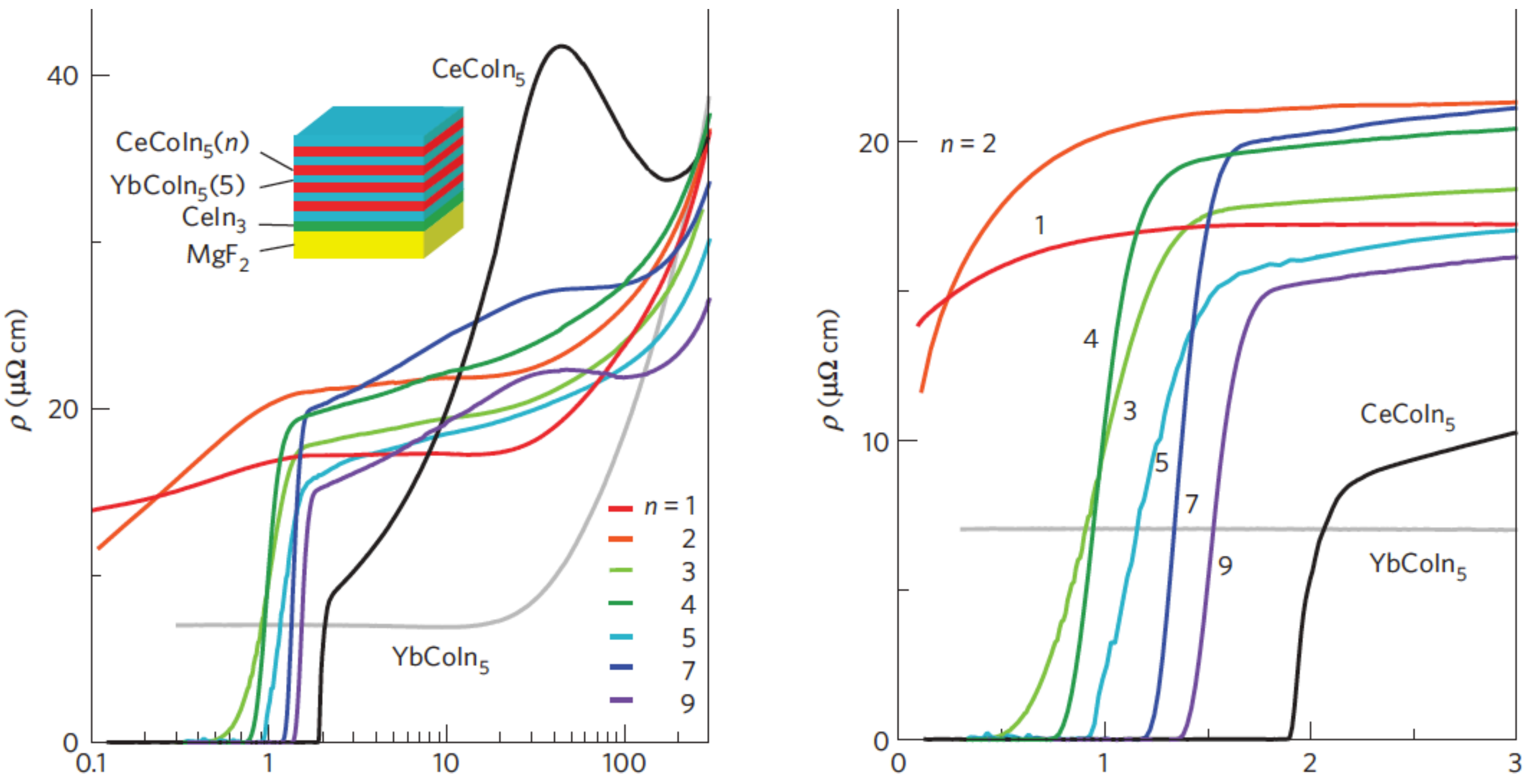
$$\frac{d}{dl}K^{-1}(l) = 4\pi^3 y^2(l),$$
$$\frac{d}{dl}y(l) = [2-\pi K(l)]y(l)$$

Correlation length: $\xi(T > T_c) \sim e^{b/\sqrt{T-T_c}}$

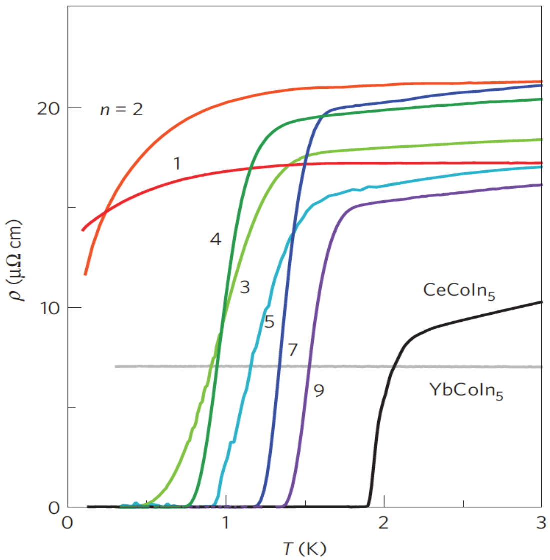
Density of unpaired vortices: $n_f \propto \xi^{-2}$

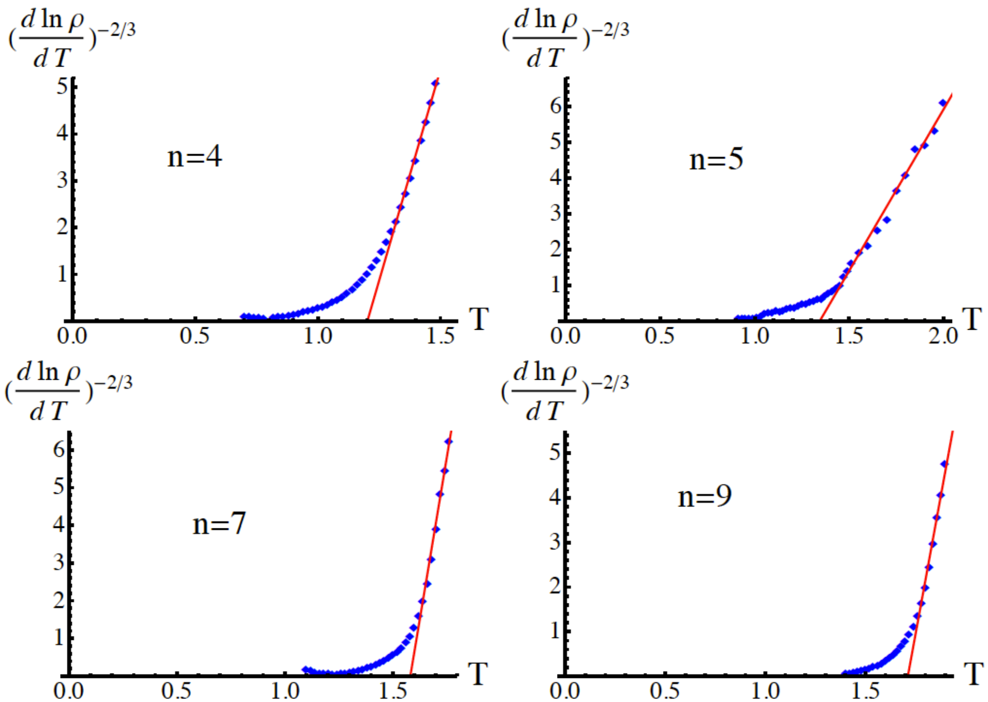
Resistivity from SC fluctuations: $\rho_{fl} \propto n_f \propto e^{-2b/\sqrt{T-T_c}}$

Check: are they BKT transitions?



Resistivity (feature III)

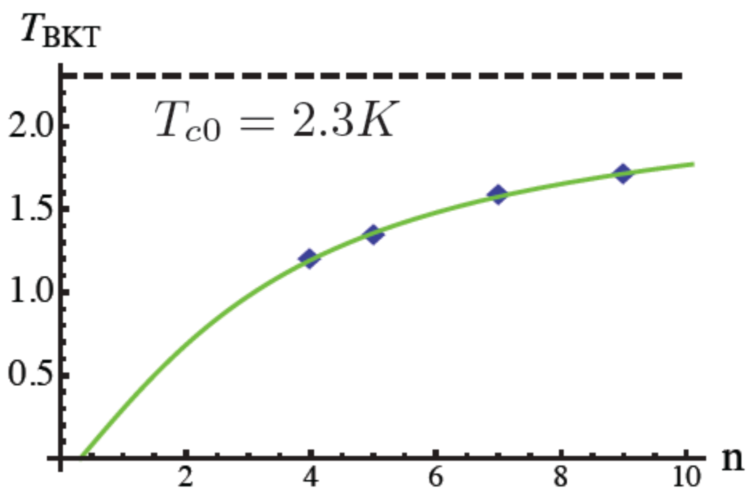


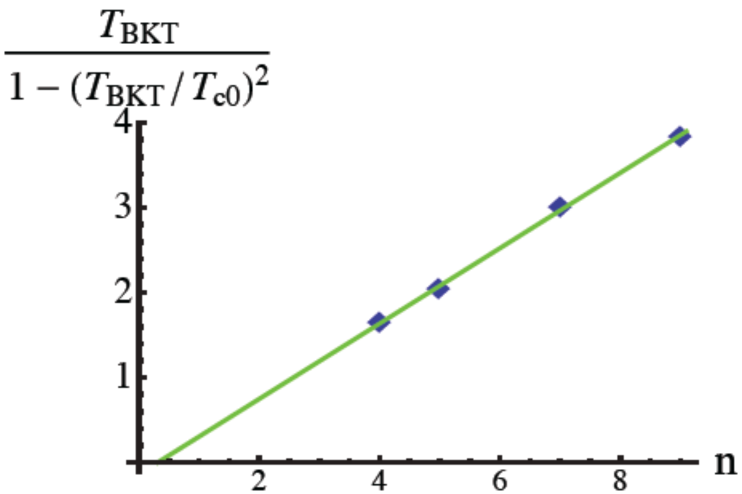


$$\rho(T) = \rho_0 e^{-b(T-T_{\mathrm{KT}})^{-1/2}}$$

$$\left[\frac{d\ln\rho(T)}{dT}\right]^{-2/3} = \left(\frac{2}{b}\right)^{2/3}(T - T_{\mathrm{KT}})$$

BKT transition temperature (feature II)

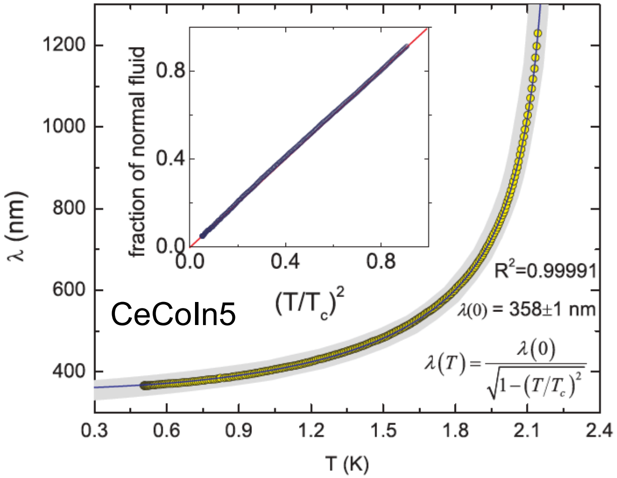




$$\frac{T_{\mathrm{BKT}}}{1-(T_{\mathrm{BKT}}/T_{c0})^2} = \frac{\pi\hbar^2 n_s^{3D}(0)}{8k_B m\epsilon_c} d$$

A large dielectric constant:

$$\epsilon_c \simeq 90$$



Kogan *et al.*, 2009

To understand the large dielectric constant

Kosterlitz's RG equation:

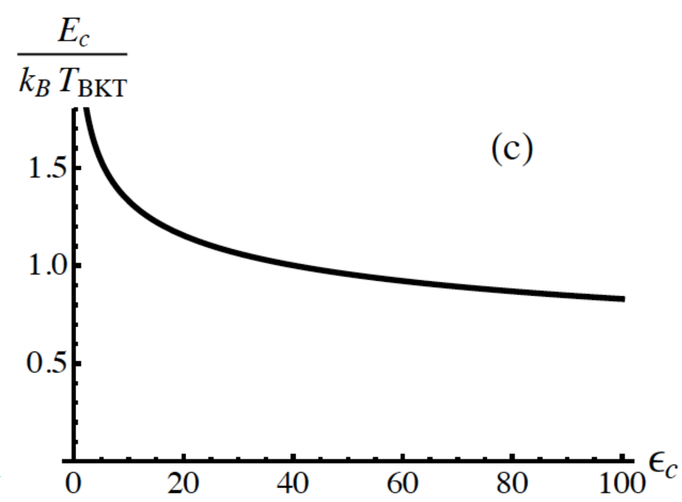
$$\frac{d}{dl}K^{-1}(l) = 4\pi^3 y^2(l),$$
$$\frac{d}{dl}y(l) = [2-\pi K(l)]y(l)$$

Dielectric constant: $$\epsilon_c = \frac{\rho_s^0(T_c^-)}{\rho_s^R(T_c^-)}$$

InOx: 1.4-1.9; YBCO: 4.6, 6;
"critical value": 1.74

Large dielectric constant means large fugacity, or small vortex core energy.

$$E_c/k_B T_{\rm BKT} \simeq (A^{1/\theta}/2\pi)\epsilon_c^{-(1-\theta)/\theta}$$
$$\theta = 0.83$$
$$A \simeq 8.62$$

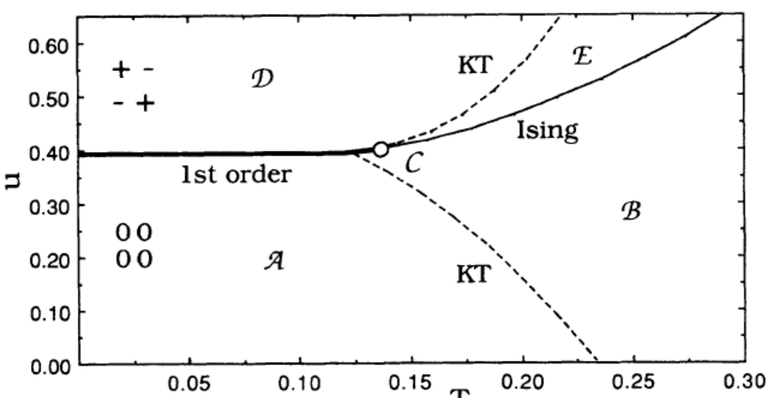


An unsolved problem:

Phase diagram of 2d vortex system at high density

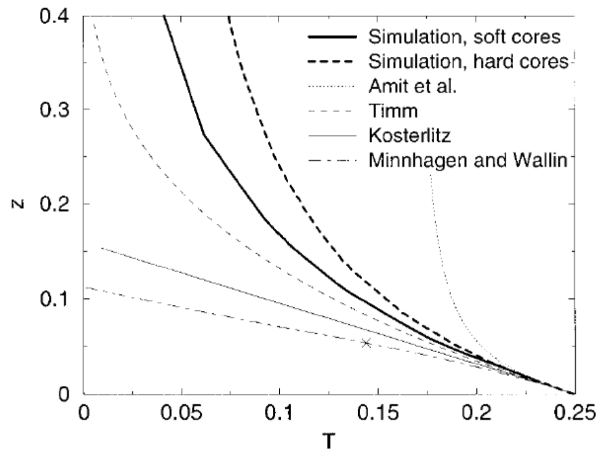
$$\frac{\mathcal{H}_v}{k_B T} = -\pi K \sum_{ij} n_i n_j \log \frac{|\mathbf{r}_i - \mathbf{r}_j|}{R_0} - \log y \sum_i n_i^2$$

Some "contradicting" numerical results:



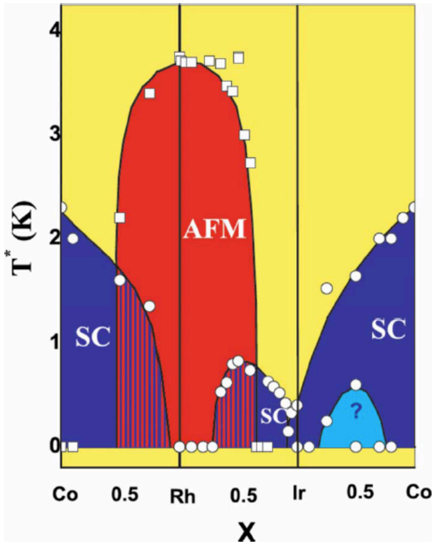
$$\mathcal{H}_{CG} = \frac{1}{2}\sum_{i,j} q_i V'(\mathbf{r}_i - \mathbf{r}_j) q_j - u \sum_i q_i^2 + \sum_i (q_i^4 - q_i^2)$$

Lee, Teitel, 1992



Lidmar, Wallin, 1997

Effect of magnetic vortex core



CeCoIn5: close to a magnetic QCP

SC vortices coupled to spin fluctuations:

$$Z[n(\boldsymbol{r})] = \int \mathcal{D}\Phi^* \int \mathcal{D}\Phi \exp\left(-\frac{\mathcal{H}_v}{k_B T} - \mathcal{S}_{sf}\right)$$

$$\frac{\mathcal{H}_v}{k_B T} = -\pi K \sum_{ij} n_i n_j \log\frac{|\mathbf{r}_i - \mathbf{r}_j|}{R_0} - \log y \sum_i n_i^2$$

$$\mathcal{S}_{sf} = \int d^2\mathbf{r} \int_0^\beta d\tau \left[\frac{1}{2}(\partial_\tau \phi + i g \mu_B \boldsymbol{H} \times \phi)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{\alpha}{2}\phi^2 + \frac{\lambda}{4}\phi^4\right]$$

$$H(\mathbf{r}) = \sum_i n_i H_0(\mathbf{r} - \mathbf{R}_i)$$

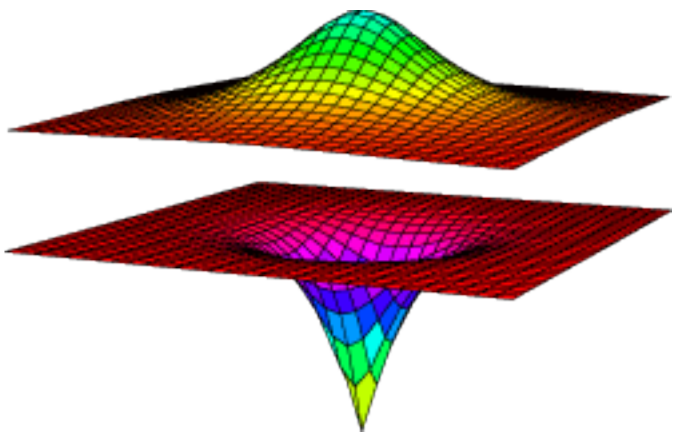
Vortex core energy

Free energy density: $$\mathcal{F}_\Phi = |\nabla\Phi|^2 + (\alpha - g^2\mu_B^2 H^2(r))|\Phi|^2 + \gamma|\Phi|^4$$

$$H_0(\mathbf{r}) \sim (\Phi_0/\lambda^2)K_0(r/\lambda)$$

Spin fluctuations reduce vortex core energy.

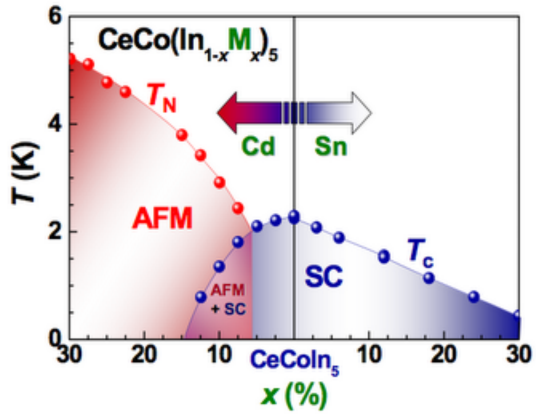
$$\delta E_c = \int d^2\mathbf{r}\mathcal{F}[\Phi(\mathbf{r})] \sim -g^4\mu_B^4\Phi_0^4/\gamma\lambda^6 \equiv -V_0 < 0.$$

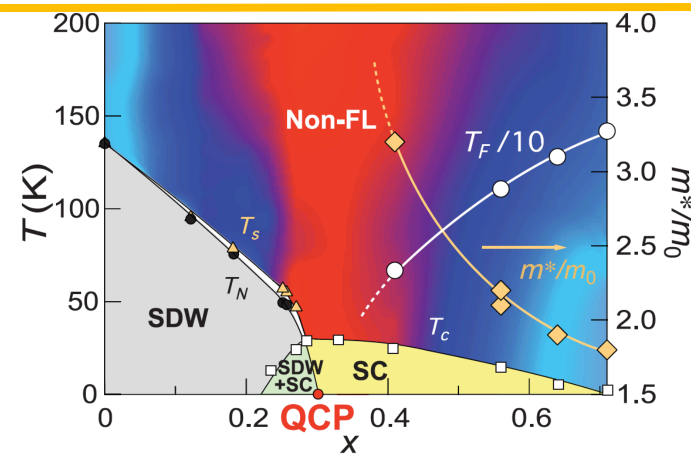


Magnetic order parameter

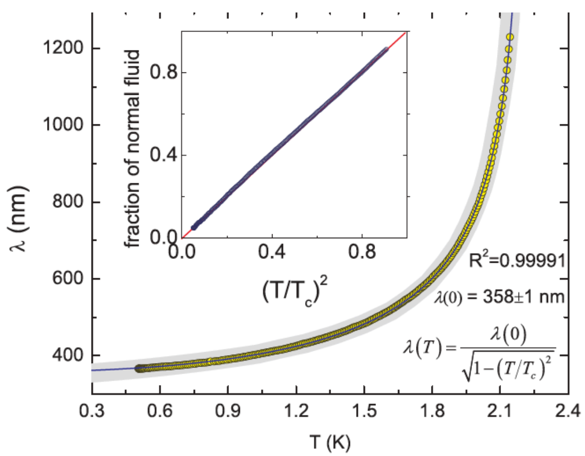
Superconducting order parameter

Superfluid density in quantum critical superconductors

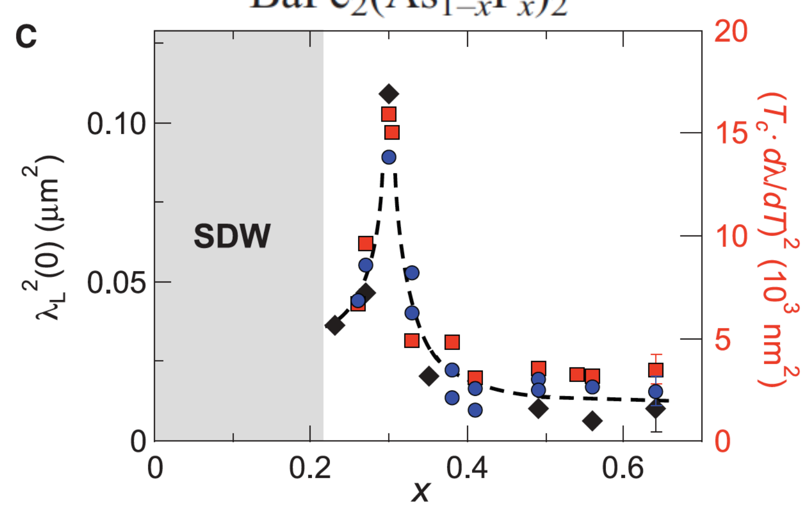




BaFe$_2$(As$_{1-x}$P$_x$)$_2$



Kogan *et al.*, 2009



Hashimoto *et al.*, 2012

Open questions for theorists:

1, Phase diagram of vortex system at high density

2, BKT transition with competing orders

3, Superfluid density in quantum critical superconductors

Is there a holographic solution?